

Response of Thermal Source in Initially Stressed Generalized Thermoelastic Half-Space with Voids

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The purpose of this research is to study the plane problem in initially stressed generalized thermoelastic half-space with voids due to thermal source. A particular type of thermal source has been taken as an application of the approach. Finite element technique has been used to solve the problem. The components of displacement, stress, temperature change and volume fraction field are computed numerically. Effect of relaxation times and initial stress are depicted graphically on the resulting quantities.

Keywords: Thermoelastic Half-Space, Voids, Initially Stressed, Thermal Sources, Finite Element Method.

1. INTRODUCTION

The study of dynamic properties of elastic solids is significant in the ultrasonic inspection of materials, vibrations of engineering structures, in seismology, geophysical and various other fields. Such materials are usually described by equations of linear elastic solids; however there are materials of a more complex microstructure (composite materials, granular materials, soils, etc.) which depict specific characteristic response to applied load. There are a number of theories which describe mechanical properties of porous materials, and one of them is a Biot¹⁻³ consolidation theory of fluid-saturated porous solids. These theories reduce to classical elasticity when the pore fluid is absent. Goodman and Cowin⁴ established a continuum theory for granular materials, whose matrix material (or skeletal) is elastic and interstices are voids. They formulated this theory from the formal arguments of continuum mechanics and introduced the concept of distributed body, which represents a continuum model for granular materials (sand, grain, powder, etc.) as well as porous materials (rock, soil, sponge, pressed powder, cork, etc.). The basic concept underlying this theory is that the bulk density of the material is written as the product of two fields, the density field of the matrix material and the volume fraction field (the ratio of the volume occupied by grains to the bulk volume at a point of the material). This representation of the bulk density of the material introduces an additional kinematic

variable in the theory. This idea of such representation of the bulk density was employed by Nunziato and Cowin⁵ to develop a non-linear theory of elastic material with voids. They developed the constitutive equations for solid like material which are non conductor of heat and discussed the restrictions imposed on these constitutive equations by thermodynamics. They showed that the change in the volume fraction causes an internal dissipation in the material which is similar to that associated with viscoelastic materials. They also considered the dynamic response and derived the general propagation conditions on acceleration waves. Later on Cowin and Nunziato⁶ developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They considered several application of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beam, and small amplitudes of acoustic waves. The small acoustic waves in an infinite elastic material with voids showed that two distinct types of longitudinal waves and a transverse wave can propagate without affecting the porosity of the material and without attenuation. The two types of longitudinal waves are attenuated and dispersed; one longitudinal wave is associated with elastic property of the material and the second associated with the property of the change in porosity of the material. These longitudinal acoustic waves are both attenuated and dispersed due to the change in the material porosity. Iesan⁷⁻⁹ has developed a linear theory of thermoelastic material with voids by generalizing some ideas of the papers. In Iesan⁸ developed a theory of initially

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stressed thermoelastic material with voids. Ciarletta and Scalia¹⁰ investigated the uniqueness and reciprocity theorems in linear thermoelasticity of material with voids.

During the last three decades, non-classical theories of thermoelasticity so called generalized thermoelasticity have been developed in order to remove the paradox of physically impossible phenomenon of infinite velocity of thermal signals in the conventional coupled thermoelasticity. Lord-Shulman¹¹ theory and Green-Lindsay¹² theory are important generalized theories of thermoelasticity that become center of interest of recent research in this area. In Lord-Shulman theory, a flux rate term into the Fourier's law of heat conduction is incorporated (with one relaxation time) and formulated a generalized theory admitting finite speed for thermal signals. Green-Lindsay theory called as temperature rate-dependent thermoelasticity in which temperature rate-dependent is included among the consecutive variables with two constants that act as two relaxation times, which does not violate the classical Fourier law of heat conduction when body under consideration has a center of symmetry. The Lord and Shulman theory of generalized thermoelasticity was further extended to homogeneous anisotropic heat conducting materials recommended by Dhaliwal and Sherief.¹³ All these theories predict a finite speed of heat propagation.

Initial stresses are developed in the medium due to many reasons, resulting from difference of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external of forces, gravity variations, etc. The Earth is supposed to be under high initial stresses. It is therefore of great interest to study the effect of these stresses on the propagation of stress waves. During the last five decades considerable attenuation has been directed towards this phenomenon. Biot³ depicted that the acoustic propagation under initial stresses would be fundamentally different from that under stress free state. He has obtained the velocities of longitudinal and transverse waves along the coordinate axis only.

Singh et al.¹⁴ discussed the reflection of generalized thermoelastic waves from a solid half-space under hydrostatic initial stress. Fahmy and El-Shahat¹⁵ studied the effect of initial stress and inhomogeneity on the thermoelastic stresses in a rotating anisotropic solid. Effect of magnetic field and initial stress on the propagation of interface waves in transversely isotropic perfectly conducting media was investigated by Acharya, Roy and Sengupta.¹⁶ Abd-Alla and Alsheikh¹⁷ studied the reflection and refraction of plane quasi-longitudinal waves at an interface of two piezoelectric media under initial stresses. Singh¹⁸ investigated the wave propagation in a prestressed piezoelectric half-space.

The exact solution of the governing equations in initially stressed thermoelastic half-space with voids for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problems. A numerical solution technique is used to calculate the solution of general

problems. For this reason the finite element method is chosen.

The finite element method is a powerful technique originally developed for numerical solution of complex problem in structural mechanics, and it remains the method of choice for complex system. A further benefit of this method is that its allow physical effects to be visualized and quantified regardless of experimental limitations. Abbas et al.^{19–23} and Othman et al.²⁴ have successfully applied finite element method to various problems in generalized thermoelastic materials. Recently,^{25–27} variants problems in waves are studied. Other forms are described for example in the Refs. [28–30].

The aim of the present study is to enhance our knowledge about the application of finite element method in initially stressed thermoelastic half-space with voids. This study has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipes and metallurgy.

In the present paper, the components of displacement, stress, temperature change and volume fraction field are obtained due to thermal source. The resulting quantities are computed numerically by finite element technique and depicted graphically.

2. BASIC EQUATION

Following Lord and Shulman,¹¹ Magana and Quintanilla,³¹ Iesan,⁸ the basic equations for homogeneous initially stressed generalized thermoelastic with voids material are

The stress-strain relation in isotropic medium

$$t_{ij} = \lambda \delta_{ij} u_{l,l} + \mu (u_{i,j} + u_{j,i}) + t^0 u_{i,j} + \xi^* \delta_{ij} \varphi - \beta \delta_{ij} \left(1 + \tau_1 \frac{\partial}{\partial t} \right) T \quad (1)$$

Equations of motion

$$(\mu + t^0) \Delta \mathbf{u} + (\lambda + \mu) \text{grad div } \mathbf{u} + \xi^* \text{grad } \varphi - \beta \left(1 + \tau_1 \frac{\partial}{\partial t} \right) \text{grad } T = \rho \ddot{\mathbf{u}} \quad (2)$$

Equilibrated stress equation of motion

$$(d\Delta - \zeta) \varphi - \omega_0 \dot{\varphi} - \xi^* \text{div } \mathbf{u} + b_1^* \left(1 + \tau_1 \frac{\partial}{\partial t} \right) T = \rho \chi \ddot{\varphi} \quad (3)$$

Equation of heat conduction

$$\left(1 + \tau_0 \frac{\partial}{\partial t} \right) \rho C^* \dot{T} + T_0 \tau_{m0} (b_1^* \dot{\varphi} + \beta \text{div } \dot{\mathbf{u}}) = k \Delta T \quad (4)$$

where

$$\tau_{m0} = 1 + m_0 \tau_0 \frac{\partial}{\partial t} \quad (5)$$

where λ , μ are Lamé's constants, ρ is the density, $\mathbf{u} = (u, v, w)$ is the displacement vector, σ_{ij} is the stress tensor,

t_{jm}^0 is the initial stress parameter, ξ^*d , b_1^* , ω_0 , ζ are the constitutive constant of the medium C^* is the specific heat, $\beta = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion. Δ is the Laplacian operator, $\varphi(= \nu - \nu_0)$ is the volume fraction field and ν_0 is the matrix volume fraction at the reference state. T is the temperature change which is measured from the absolute temperature T_0 ($T_0 \neq 0$). We assume that T_0 and ν_0 are constants. τ_0 , τ_1 are thermal relaxation times. For Lord and Shulman (LS) theory $m_0 = 1$ and for Green and Lindsay (GL) theory $m_0 = 0$. The thermal relaxation times τ_0 and τ_1 satisfies the inequality $\tau_1 \geq \tau_0 > 0$ for GL-theory only. However, it has been proved by Sturnin³² that the inequality is not mandatory for τ_0 and τ_1 to follow. In the above equations the comma notation denotes partial derivatives with respect to special coordinate and dot denotes the derivative with respect to time.

2.1. Formulation of the Problem

We consider the medium of isotropic generalized thermoelastic diffusion with voids under initial stress. The origin of the Cartesian coordinate system (x, y, z) is taken at any point and z -axis taking vertically downward into the medium. For two dimensional problem, we have

$$\mathbf{u} = (u, 0, w) \quad (6)$$

We define the dimensionless quantities:

$$\begin{aligned} x' &= \frac{\omega_1^* x}{c_1}, \quad z' = \frac{\omega_1^* z}{c_1}, \quad t' = \omega_1^* t, \quad u' = \frac{\omega_1^* u}{c_1}, \quad w' = \frac{\omega_1^* w}{c_1}, \\ \varphi' &= \frac{\omega_1^{*2} \chi \varphi}{c_1^2}, \quad T' = \frac{\beta T}{\rho c_1^2}, \quad t_{ij}' = \frac{t_{ij}}{\beta T_0}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad (7) \\ \tau_0' &= \omega_1^* \tau_0, \quad \tau_1' = \omega_1^* \tau_1, \quad \omega_1^* = \frac{C^*(\lambda + 2\mu)}{k} \end{aligned}$$

Here ω_1^* and c_1 are the characteristic frequency and longitudinal wave velocity in the medium respectively.

Upon introducing the quantities (7) in the Eqs. (2)–(5) with the aid of (6) and after suppressing the primes, we obtain

$$\delta_1 \Delta u + \delta_2 \frac{\partial e}{\partial x} + \delta_3 \frac{\partial \varphi}{\partial x} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial x} = \ddot{u} \quad (8)$$

$$\delta_1 \Delta w + \delta_2 \frac{\partial e}{\partial z} + \delta_3 \frac{\partial \varphi}{\partial z} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial z} = \ddot{w} \quad (9)$$

$$\left(\delta_4 \Delta + \delta_5 + \delta_6 \frac{\partial}{\partial t}\right) \varphi + \delta_7 e + \delta_8 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T = \ddot{\varphi} \quad (10)$$

$$\tau_{m0} (\delta_9 \dot{e} + \delta_{10} \dot{\varphi}) + \delta_{11} \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \dot{T} = k \Delta T \quad (11)$$

where

$$\delta_1 = \frac{t^0 + \mu}{\lambda + 2\mu}, \quad \delta_2 = \frac{\lambda + \mu}{\lambda + 2\mu}, \quad \delta_3 = \frac{\xi^*}{\chi \rho \omega_1^{*2}},$$

$$\begin{aligned} \delta_4 &= \frac{d}{\chi(\lambda + 2\mu)}, \quad \delta_5 = -\frac{\zeta}{\rho \chi \omega_1^{*2}}, \quad \delta_6 = -\frac{\omega_0}{\rho \chi \omega_1^*}, \\ \delta_7 &= -\frac{\xi^*}{\lambda + 2\mu}, \quad \delta_8 = \frac{b_1^*}{\beta}, \quad \delta_9 = \frac{T_0 \beta^2}{\rho \omega_1^*}, \quad (12) \\ \delta_{10} &= \frac{T_0 b_1^* c_1^2 \beta}{\rho \chi \omega_1^{*3}}, \quad \delta_{11} = \frac{\rho C^* c_1^2}{\omega_1^*}, \quad \delta_{12} = \frac{B c_1^2}{\chi \omega_1^{*2}}, \\ e &= \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}, \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \end{aligned}$$

From the Eqs. (1) and (7), the stress components in dimensionless form are,

$$\begin{aligned} t_{xx} &= \frac{1}{\beta T_0} \left[(\lambda + 2\mu + t^0) u_{,x} + \lambda w_{,z} \right. \\ &\quad \left. + \delta_{12} \varphi - \rho c_1^2 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T \right] \quad (13) \end{aligned}$$

$$\begin{aligned} t_{zz} &= \frac{1}{\beta T_0} \left[\lambda u_{,x} + (\lambda + 2\mu + t^0) w_{,z} \right. \\ &\quad \left. + \delta_{12} \varphi - \rho c_1^2 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T \right] \quad (14) \end{aligned}$$

$$t_{xz} = \frac{1}{\beta T_0} [(\mu + t^0) u_{,z} + \mu w_{,x}] \quad (15)$$

2.2. Initial and Boundary Condition

The above Eqs. (8)–(11) are solved subjected to initial conditions

$$u = w = \varphi = T = 0, \quad \dot{u} = \dot{w} = \dot{\varphi} = \dot{T} = 0, \quad t = 0 \quad (16)$$

The boundary condition for the problem may be taken a

$$\begin{aligned} T(0, z, t) &= T_0 \delta(t) \delta(2l - |z|), \quad \frac{\partial \varphi}{\partial x} = 0, \\ \sigma_{xx}(0, z, t) &= 0, \quad \sigma_{xz}(0, z, t) = 0 \end{aligned} \quad (17)$$

where $H()$ is the Heaviside unit step.

2.3. Finite Element Formulation

In this section, the governing equations of generalized thermoelastic diffusion with relaxation times are summarized, followed by the corresponding finite element equations. In the finite element method, the displacement components u , w , volume fraction φ and temperature change T are related to the corresponding nodal values by

$$\begin{aligned} u &= \sum_{i=1}^m N_i u_i(t), \quad w = \sum_{i=1}^m N_i w_i(t) \\ T &= \sum_{i=1}^m N_i T_i(t), \quad \varphi = \sum_{i=1}^m N_i \varphi_i(t) \end{aligned} \quad (18)$$

where m denotes the number of nodes per element, and N_i are the shape functions. The eight-node isoparametric, quadrilateral element is used for displacement components, volume fractional field and temperature calculations.

The weighting functions and the shape functions coincide. Thus,

$$\begin{aligned}\delta u &= \sum_{i=1}^m N_i \delta u_i, & \delta w &= \sum_{i=1}^m N_i \delta w_i \\ \delta \varphi &= \sum_{i=1}^m N_i \delta \varphi_i, & \delta T &= \sum_{i=1}^m N_i \delta T_i\end{aligned}\quad (19)$$

It should be noted that appropriate boundary conditions associated with the governing Eqs. (8)–(11) must be adopted in order to properly formulate a problem. Boundary conditions are either essential (or geometric) or natural (or traction) types. Essential conditions are prescribed displacements u, w , volume fraction φ and temperature change T while, the natural boundary conditions are prescribed tractions, heat flux and equilibrated stress which are expressed as

$$\begin{aligned}\sigma_{xx}n_x + \sigma_{xz}n_z &= \bar{\tau}_x, & \sigma_{xz}n_x + \sigma_{zz}n_z &= \bar{\tau}_z, \\ q_x n_x + q_z n_z &= \bar{q}, & h_x n_x + h_z n_z &= \bar{h}\end{aligned}\quad (20)$$

where n_x and n_z are direction cosines of the outward unit normal vector at the boundary, $\bar{\tau}_x, \bar{\tau}_z$ are the given tractions values, \bar{q} is the given surface heat flux and \bar{h} is the given equilibrated stress value. In the absence of body force, the governing equations are multiplied by weighting functions and then are integrated over the spatial domain Ω with the boundary Γ . Applying integration by parts and making use of the divergence theorem reduce the order of the spatial derivatives and allows for the application of the boundary conditions. Thus, the finite element equations corresponding to Eqs. (8)–(11) can be obtained as

$$\sum_{e=1}^m \left(\begin{bmatrix} M_{11} & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & 0 \\ 0 & 0 & 0 & M_{44} \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{w} \\ \ddot{\varphi} \\ \ddot{T} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & R_{33} & 0 \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{w} \\ \dot{\varphi} \\ \dot{T} \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ 0 & 0 & 0 & K_{44} \end{bmatrix} \begin{Bmatrix} u \\ w \\ \varphi \\ T \end{Bmatrix} \right) = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} \quad (21)$$

where the coefficients in Eq. (21) are given below

$$\begin{aligned}M_{11} &= \int_{\Omega} [N]^T [N] d\Omega, & M_{22} &= \int_{\Omega} [N]^T [N] d\Omega, \\ M_{33} &= \int_{\Omega} [N]^T [N] d\Omega, & M_{44} &= \int_{\Omega} \delta_{12} \tau_0 [N]^T [N] d\Omega,\end{aligned}$$

$$\begin{aligned}R_{33} &= \int_{\Omega} \delta_6 [N]^T [N] d\Omega, & R_{44} &= \int_{\Omega} \delta_8 [N]^T [N] d\Omega, \\ R_{41} &= \int_{\Omega} \delta_{10} [N]^T \left[\frac{\partial N}{\partial x} \right] d\Omega, & R_{42} &= \int_{\Omega} \delta_{10} [N]^T \left[\frac{\partial N}{\partial z} \right] d\Omega, \\ R_{43} &= \int_{\Omega} \delta_{11} [N]^T N d\Omega, \\ K_{11} &= \int_{\Omega} (\delta_1 + \delta_2) \left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial x} \right] + \delta_1 \left[\frac{\partial N}{\partial z} \right]^T \left[\frac{\partial N}{\partial z} \right] d\Omega, \\ K_{12} &= \int_{\Omega} \delta_2 \left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial z} \right] d\Omega, \\ K_{13} &= \int_{\Omega} \delta_4 [N]^T \left[\frac{\partial N}{\partial x} \right] d\Omega, & K_{14} &= \int_{\Omega} [N]^T \left[\frac{\partial N}{\partial x} \right] d\Omega, \\ K_{21} &= \int_{\Omega} \delta_2 \left[\frac{\partial N}{\partial z} \right]^T \left[\frac{\partial N}{\partial x} \right] d\Omega, \\ K_{22} &= \int_{\Omega} (\delta_1 + \delta_2) \left[\frac{\partial N}{\partial z} \right]^T \left[\frac{\partial N}{\partial z} \right] + \delta_1 \left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial x} \right] d\Omega, \\ K_{23} &= \int_{\Omega} \delta_3 [N]^T \left[\frac{\partial N}{\partial z} \right] d\Omega, & K_{24} &= - \int_{\Omega} [N]^T \left[\frac{\partial N}{\partial z} \right] d\Omega, \\ K_{31} &= \int_{\Omega} \delta_7 [N]^T \left[\frac{\partial N}{\partial x} \right] d\Omega, & K_{32} &= - \int_{\Omega} \delta_7 [N]^T \left[\frac{\partial N}{\partial z} \right] d\Omega, \\ K_{33} &= \int_{\Omega} \left(\delta_4 \left[\left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial x} \right] + \left[\frac{\partial N}{\partial z} \right]^T \left[\frac{\partial N}{\partial z} \right] \right) + \delta_5 [N]^T N \right) d\Omega, \\ K_{34} &= \int_{\Omega} \delta_4 [N]^T N d\Omega, \\ K_{44} &= \int_{\Omega} k \left(\left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial x} \right] + \left[\frac{\partial N}{\partial z} \right]^T \left[\frac{\partial N}{\partial z} \right] \right) d\Omega, \\ F_1 &= \int_{\Gamma} [N]^T \bar{\tau}_x d\Gamma, & F_2 &= \int_{\Gamma} [N]^T \bar{\tau}_y d\Gamma, \\ F_3 &= \int_{\Gamma} [N]^T \bar{q} d\Gamma, & F_4 &= \int_{\Gamma} [N]^T \bar{h} d\Gamma\end{aligned}\quad (22)$$

Symbolically, the discretized equations of Eqs. (21) can be written as

$$M\ddot{d} + R\dot{d} + Kd = F^{\text{ext}} \quad (23)$$

where M , R , K and F^{ext} represent the mass, damping, stiffness matrices and external force vectors, respectively; $d = [u \ w \ \varphi \ T]^T$. On the other hand, the time derivatives of the unknown variables have to be determined by Newmark time integration method (see Wriggers Ref. [33]).

2.4. Numerically Results and Discussion

With the view of illustrating theoretical results derived in the preceding sections, and compare these in the context of various theories of initially stressed thermoelastic with voids, we now present some numerical results for copper material, the physical data for which is given by

$$\begin{aligned}\lambda &= 7.76 \times 10^{10} \text{ Kg m}^{-1} \text{ s}^{-2}, & \mu &= 3.86 \times 10^{10} \text{ Kg m}^{-1} \text{ s}^{-2}, \\ k &= 386 \text{ W m}^{-1} \text{ K}^{-1}, & \rho &= 8.954 \times 10^3 \text{ Kg m}^{-3}\end{aligned}$$

$$\alpha_t = 1.78 \times 10^{-5} \text{ K}^{-1}, \quad C^* = 0.3831 \times 10^3 \text{ J Kg}^{-1} \text{ K}^{-1},$$

$$T_0 = 0.293 \times 10^3 \text{ Kg m}^{-3}$$

The voids parameter, initial stress parameters and relaxation times are

$$\omega_0 = 0.687 \times 10^{-5} \text{ N s m}^{-2}, \quad \chi = 0.610 \times 10^{-15} \text{ m}^2,$$

$$d = 0.9798 \times 10^{-5} \text{ N}, \quad \xi^* = 0.4 \times 10^{10} \text{ N m}^{-2},$$

$$t^0 = 0.5 \times 10^{10} \text{ N m}^{-2}, \quad \tau_0 = 0.02 \text{ s}, \quad \tau_1 = 0.05 \text{ s},$$

$$\zeta = 0.196 \times 10^4 \text{ N m}^{-2}$$

3. DISCUSSION

Case 1: Figures 1–7 shows the variation of normal displacement u and tangential displacement w , Temperature change T , volume fraction field ϕ , Normal and stress components t_{xx} , t_{xz} , t_{zz} with x for CT, LS and GL theories of thermoelasticity. In these figures solid line, dash line and dotted lines corresponds to CT, LS and GL theories of thermoelasticity.

Figure 1 shows that value of u increase initially and in this range the value of u are higher for LS and minimum for GL theory and thereafter decrease with increase in x and shows the opposite behavior. The values of CT theory lies between the values of LS and GL theories.

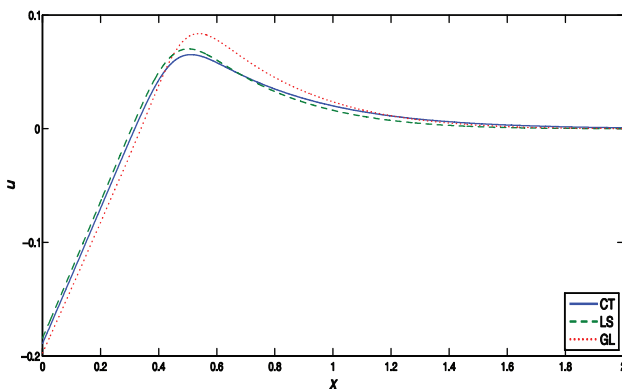


Fig. 1. The horizontal displacement distribution u under three theories.

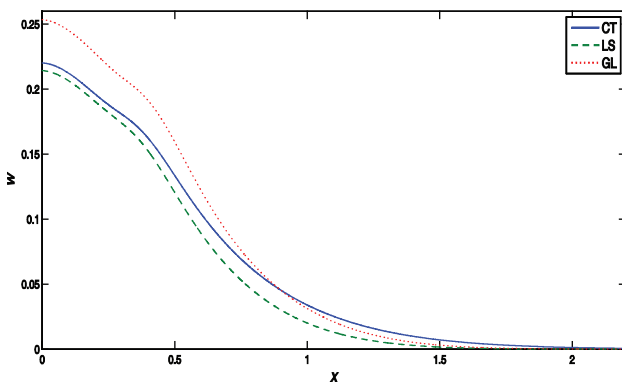


Fig. 2. The vertical displacement distribution w under three theories.

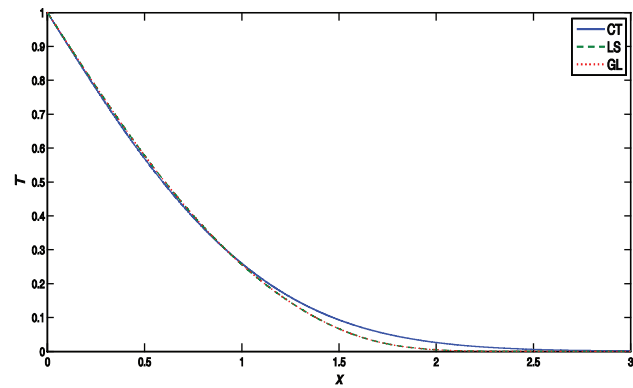


Fig. 3. The temperature distribution T under three theories.

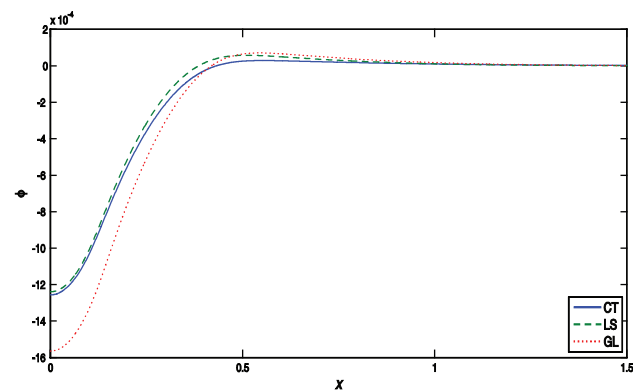


Fig. 4. The volume fraction ϕ distribution under three theories.

Figure 2 shows that tangential displacement w decrease when $0 < x < 1.5$ and has a constant behavior for $x > 1.5$. In the range $0 < x < 0.8$, w attains higher value for GL theory and for $x > 0.8$, the higher values occurs for CT theory. The minimum values occurs for LS theory for all values of x .

Figure 3 indicates that behavior of temperature change T is similar for all theories CT, LS, GL in the range $0 < x < 1$ and CT attains maximum value as x increases further. Temperature change decrease with increase in x and

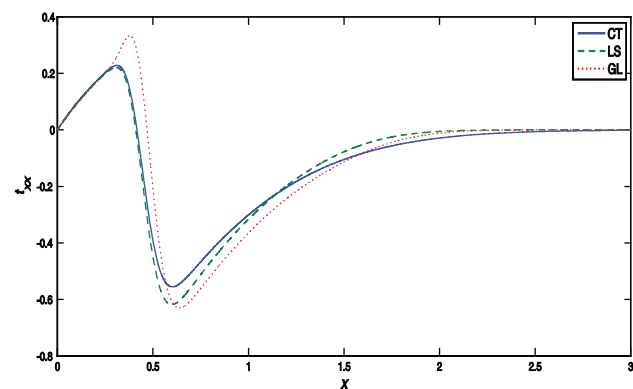


Fig. 5. The stress t_{xx} distribution under three theories.

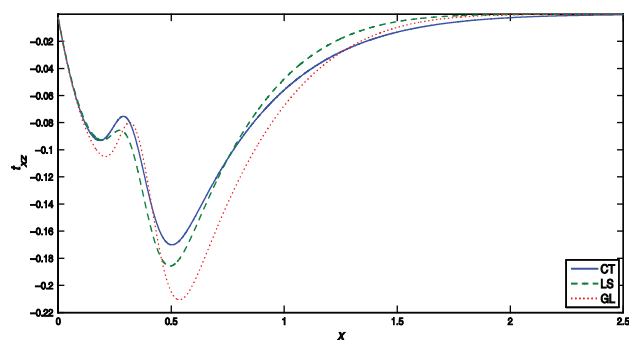


Fig. 6. The stress t_{xz} distribution under three theories.

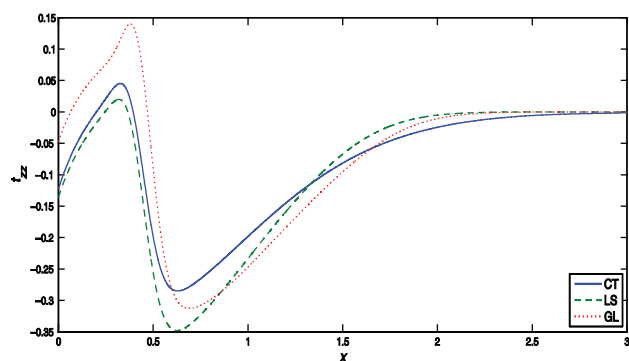


Fig. 7. The stress t_{zz} distribution under three theories.

becomes constant as x increases further for all theories of thermoelasticity.

Figure 4 shows that the value of volume fraction field ϕ first increase smoothly with x and then becomes constant. Initially maximum value occurs for LS theory and minimum value for GL theory in the range $0 < x < 0.5$ and shows opposite behavior for $0.5 < x < 1$. The behavior of values of CT theory is similar and its values lies between the values of LS and GL theories when $0 < x < 0.4$ and after that its value are smaller than the LS and GL theories.

Figure 5–7 depicts that stress components t_{xx} , t_{zz} and t_{xz} fluctuates in the range $0 < x < 1.5$ and increase in the range $1.5 < x < 2.5$ and constant behavior is noticed

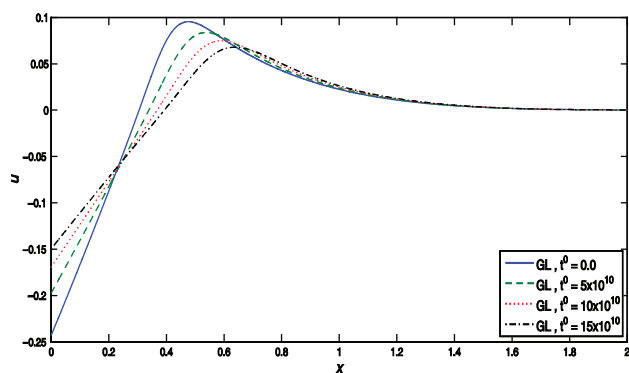


Fig. 8. The horizontal displacement distribution u for different values of initial stress t^0 under GL theory.

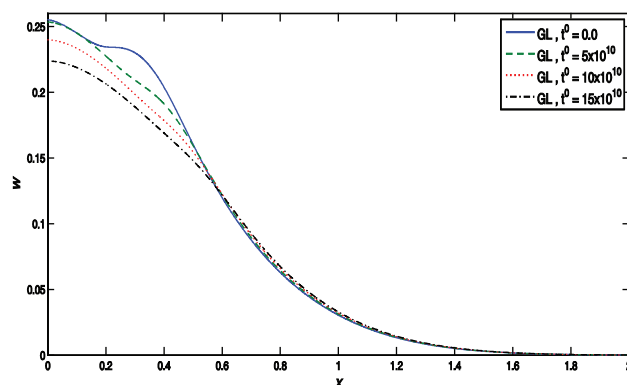


Fig. 9. The vertical displacement distribution w for different values of initial stress t^0 under GL theory.

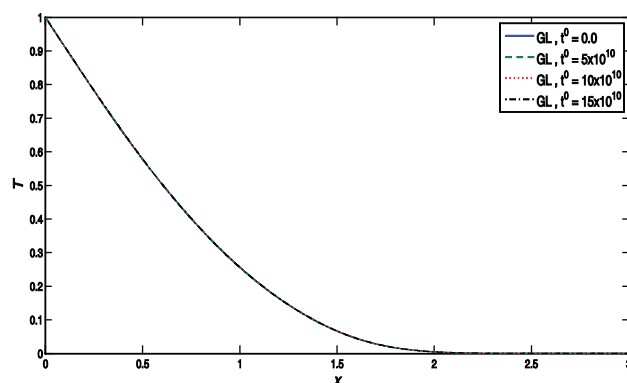


Fig. 10. The temperature T distribution for different values of initial stress t^0 under GL theory.

elsewhere. For GL theory, t_{xx} attains extreme values and t_{zz} attains maximum value. Also t_{zz} attains minimum value for LS theory at $x = 0.6$. The behavior and variation of t_{xx} , t_{zz} and t_{xz} are similar for CT theories as for LS and GL theories with difference in their magnitude values.

Case 2: Figures 8–14 shows the variation of Normal displacement u and tangential displacement w , Temperature

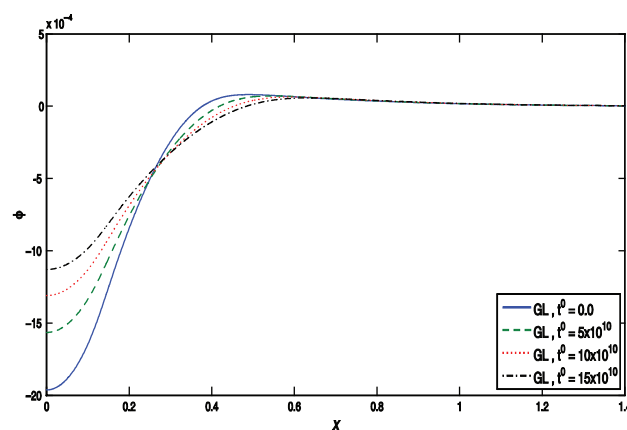


Fig. 11. The volume fraction ϕ distribution for different values of initial stress t^0 under GL theory.

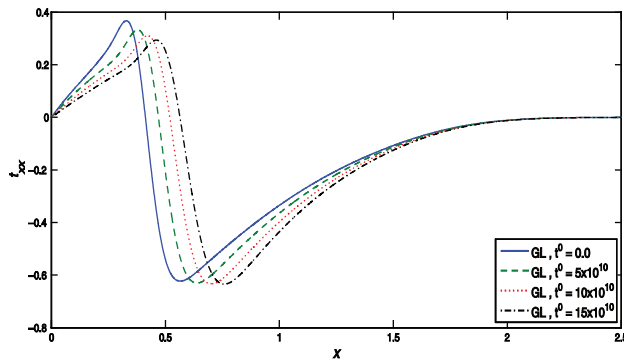


Fig. 12. The stress t_{xx} distribution for different values of initial stress t^0 under GL theory.

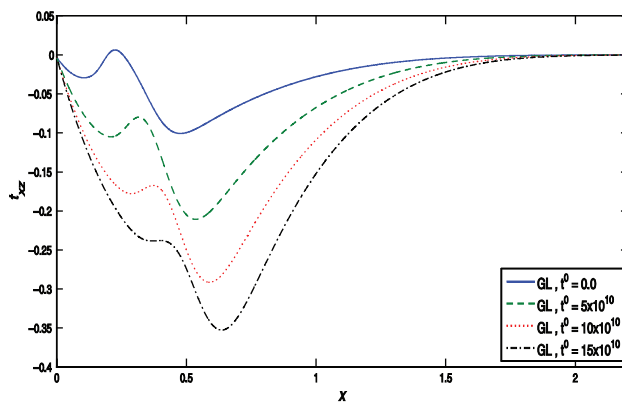


Fig. 13. The stress t_{xz} distribution for different values of initial stress t^0 under GL theory.

change T , volume fraction field φ , stress components t_{xx} , t_{xz} and t_{zz} with x for different initial stress parameters. In these figures solid line, dash line, dotted line and dash line with dots corresponds to initial stress parameters $t^0 = 0.5 \times 10^{10}$, 10×10^{10} , 15×10^{10} respectively.

Figure 8 shows that the value of u increase rapidly in the range $0 < x < 0.6$ and then decrease and becomes constant. In the range $0 < x < 0.25$, the values of normal displacement u increase with increase in relaxation times.

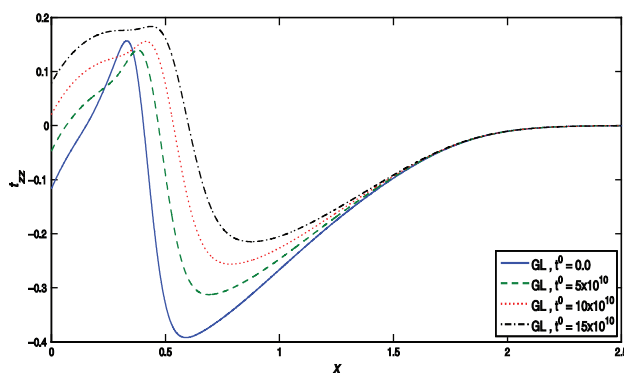


Fig. 14. The stress t_{zz} distribution for different values of initial stress t^0 under GL theory.

The behavior of u for all initial stress parameters are opposite in the ranges $0 < x < 0.25$ and $0.25 < x < 0.6$.

It is noticed from Figure 9 that tangential displacement (w) decrease in the range $0 < x < 0.6$ and its value decreases as initial stress parameter increases with maximum value occurring for $t^0 = 0$ and for $x > 0.6$ it decrease continuously for all initial stress parameters and shows negligible variation for different initial stress parameters and finally constant behavior is noticed.

Figure 10 shows that temperature change T coincide for all initial stress parameters and decrease smoothly and becomes constant for $x > 2$.

Figure 11 shows that φ increase smoothly in the range $0 < x < 0.6$ and then becomes constant. The behavior of and variation in values of φ are opposite in the ranges $0 < x < 0.25$ and $0.25 < x < 0.6$ for all initial stress parameters.

Figures 12–14 depicts that stress components t_{xx} , t_{zz} and t_{xz} fluctuates in the range $0 < x < 1.5$ and increase in the range $1.5 < x < 2.5$ and becomes constant elsewhere. t_{xx} attains maximum value for $t^0 = 0$ whereas point at which the minimum value of t_{xx} occurs is same for all initial stress parameters. The values of t_{xz} decrease with increase in initial stress parameter whereas values of t_{zz} increase with increase in initial stress parameter.

4. CONCLUSION

The plane problem in initially stressed generalized thermoelastic half-space with voids due to thermal source is studied for a particular type of thermal source using finite element technique. The components of displacement, stress, temperature change and volume fraction field are computed numerically. are depicted graphically on the resulting quantities. Appreciable effect of initial stress parameter is observed on the resulting quantities.

The variation of normal and tangential displacement u , w , Temperature change T , volume fraction field φ , stress components t_{xx} , t_{xz} , t_{zz} for CT, LS, GL theories of thermoelasticity is significant.

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